JEE MAIN 2026

Sample Paper – 9

Time Allowed: 3 hours **Maximum Marks:** 300

General Instructions:

1. The test consists of total **75 questions.**

2. Each subject **(PCM)** has **25 questions.**

3. Each subject divided into two sections. **Section A** consists of 20 multiple-choice questions & **Section B** consists of 5 numerical value-type questions.

4. Marking Scheme:

- **Section A (MCQs):** +4 marks for each correct answer, –1 mark for each incorrect answer, 0 marks for unattempted.
- **Section B (Numerical):** +4 marks for each correct answer, 0 marks for incorrect or unattempted.

5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.

6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

Physics

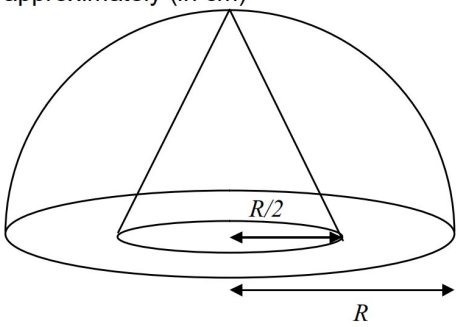
PART – A

SECTION – A

(One Options Correct Type)

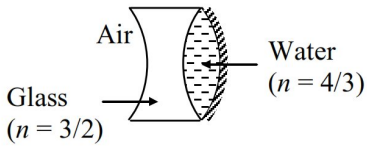
*This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.*

1. From a uniform solid hemisphere of radius R, a solid cone of base radius R/2 and height R is cut out as shown in figure. If R=5cm, then the height of the centre of mass of the remaining object is approximately (in cm)



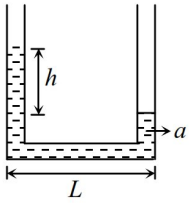
(A) 2
(B) 3
(C) 3/8
(D) none of these

2. The radius of curvature of the left and right surface of the concave lens are 10 cm and 15 cm respectively. The radius of curvature of the mirror is 15 cm



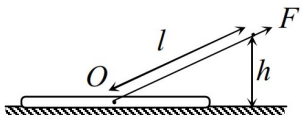
(A) equivalent focal length of the combination is –18 cm
(B) equivalent focal length of the combination is +36 cm
(C) the system behaves like a concave lens
(D) the system behaves like a convex mirror

3. At rest, a liquid stands at the same level in the tubes. As the system is given an acceleration *a* towards the right, a height difference *h* occurs as shown in the figure. The value of *h* is:



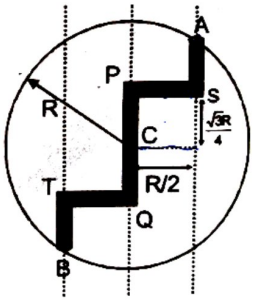
(A) $\frac{aL}{2g}$
(B) $\frac{gL}{2a}$
(C) $\frac{gL}{a}$
(D) $\frac{aL}{g}$

4. A log of mass *m* is pulled at a constant velocity and with a force *F* by means of a rope of length *l*. The distance between the end of the rope and the ground is *h* as shown. The co-efficient of friction between the log and the ground is



(A) $\frac{F\sqrt{l^2-h^2}}{mgl-Fh}$
(B) $\frac{F\sqrt{l^2-h^2}}{mgl+Fh}$
(C) $\frac{Fh}{mgl-F\sqrt{l^2-h^2}}$
(D) $\frac{Fh}{mgl+F\sqrt{l^2-h^2}}$

5. A zigzag tunnel ASPCQTB is made in earth as shown in diagram. A particle is released from A and all the collisions are perfectly inelastic. Find out the time to reach from A to C.



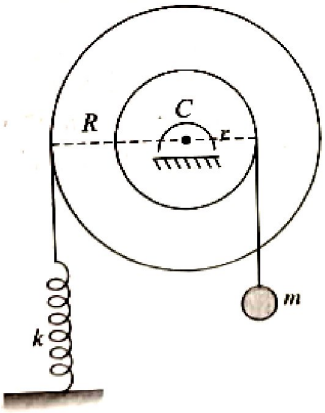
(A) $\frac{3\pi}{2}\sqrt{\frac{Re}{g}}$

(B) $2\pi\sqrt{\frac{Re}{g}}$

(C) $\frac{4\pi}{3}\sqrt{\frac{Re}{g}}$

(D) Never reach

6. A stepped disc of mass M and radius R is pivoted at its center C smoothly. An inextensible string connected with a light spring of stiffness k passes over the pulley. One end of the string is rigidly connected with the ground and the other end is attached to a body of mass m. If the string does not slide on the pulley, find the angular frequency of oscillation of the system.



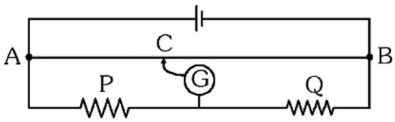
(A) $\omega = \sqrt{\frac{kR^2}{MR^2 + 2mr^2}}$

(B) $\omega = \sqrt{\frac{2kR^2}{mr^2 + 2MR^2}}$

(C) $\omega = \sqrt{\frac{2kR^2}{MR^2 + 2mr^2}}$

(D) $\omega = \sqrt{\frac{4kR^2}{MR^2 + 2mr^2}}$

7. The 'wire AB' is now a part of the adjacent circuit. With the resistors P = 50 W and Q = 100 W, the null point is obtained at C where AC = 33 cm. When the resistors are interchanged, the null point is found at C with AC = 67 cm. The systematic error in this experiment seems to be due to non-coincidence of A and B with 0 cm mark and 100 cm mark respectively. If these end errors and equivalent to 'a' cm and 'b' cm respectively, then they are



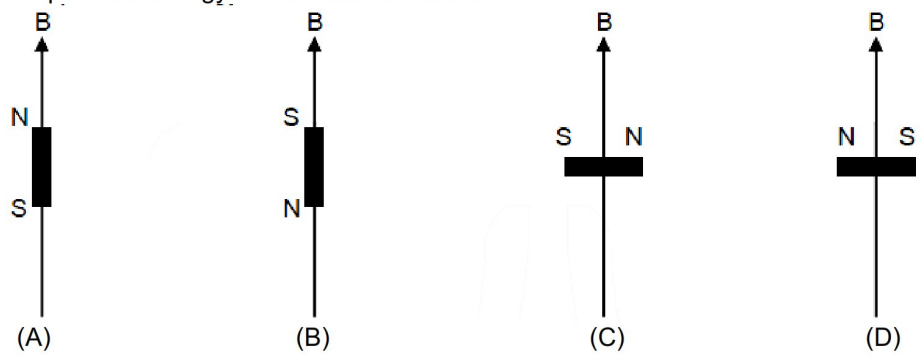
(A) 0 and 1

(B) 1 and 0

(C) 0.33 and 0.33

(D) 1 and 1

8. Consider different orientations of a bar magnet lying in a uniform magnetic field as shown below. The potential energy is maximum in orientation



(A) (B) (C) (D)

9. A conducting wire is bent in the form of a n sided regular polygon enclosed by a circle of radius R. The magnetic field produced at its centre by a current i flowing through the wire is

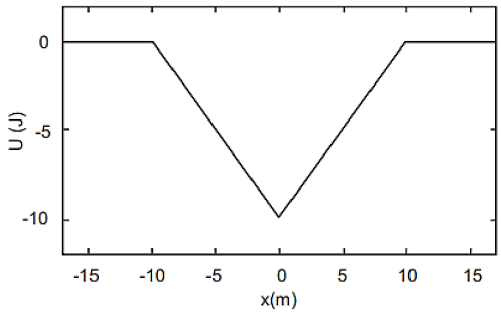
(A) $\frac{\mu_0 i}{2R}\frac{\sin\frac{\pi}{n}}{\frac{\pi}{n}}$

(B) $\frac{\mu_0 i}{2R}\frac{\cos\frac{\pi}{n}}{\frac{\pi}{n}}$

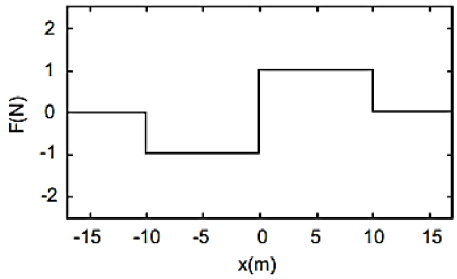
(C) $\frac{\mu_0 i}{2R}\frac{\tan\frac{\pi}{n}}{\frac{\pi}{n}}$

(D) $\frac{\mu_0 i}{2R}\frac{\cot\frac{\pi}{n}}{\frac{\pi}{n}}$

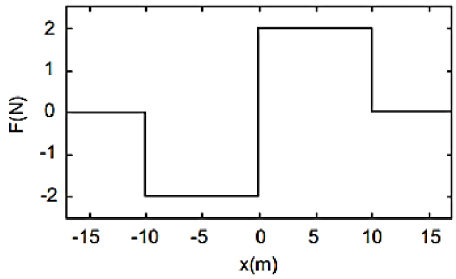
10. The potential energy (U) of a particle moving in a potential field varies with its displacement (x) as shown below.



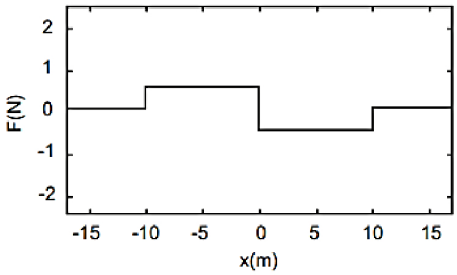
The variation of force F(x) acting on the particle as a function of x can be represented by



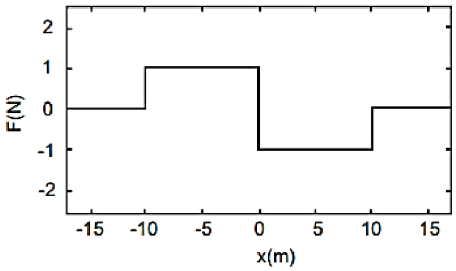
(Fig i)



(Fig ii)



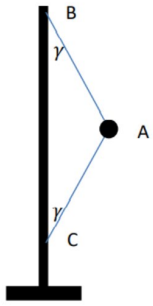
(Fig iii)



(Fig iv)

(A) Fig (i)
(B) Fig (ii)
(C) Fig (iii)
(D) Fig (iv)

11. The strings AB and AC each of length 40 cm, connect a ball of mass 200 g to a vertical shaft as shown. When the shaft rotates at a constant angular speed ω, the ball travels in a horizontal circle with the strings inclined at $\gamma = 30^\circ$ to the shaft. If the tension in the string AC is 4 N, that in the string AB and the angular speed ω respectively, are

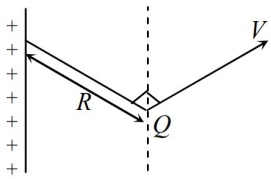


(A) 6.26 N and 11.32 rad/s
(B) 7.92 N and 14.32 rad/s
(C) 7.92 N and 11.32 rad/s
(D) 6.26 N and 14.32 rad/s

12. An alpha particle with kinetic energy K approaches a stationary nucleus having atomic number Z. The distance of closest approach is b. Therefore the distance of closest approach for a nucleus of atomic number 2Z is

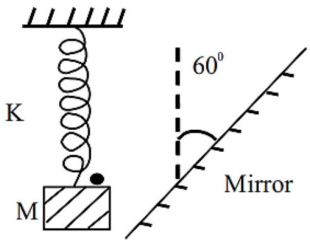
(A) b/2
(B) $\sqrt{2}$b
(C) 2 b
(D) 4 b

13. A point charge +*Q* is projected with velocity *V* from a distance *R* from an infinitely long fixed line charge of linear charge density λ. What is the tangential velocity of the point charge when its radial distance from the line charge becomes *eR* ?



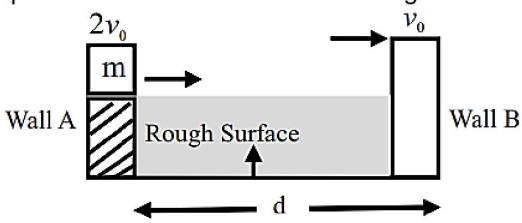
(A) $\frac{V}{e}$
(B) eV
(C) e^2V
(D) V/e^2

14. An insect of negligible mass is sitting on a block of mass M, tied with a spring of force constant K. The block performs simple harmonic motion vertically with amplitude A in front of a mirror which is inclined at 60° with the vertical as shown. The maximum speed of insect relative to its image will be



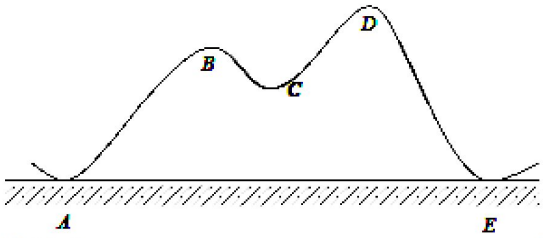
(A) $2A\sqrt{\frac{K}{M}}$
(B) $A\sqrt{\frac{3K}{M}}$
(C) $A\sqrt{\frac{K}{M}}$
(D) zero

15. As shown in figure, a block of mass m is projected from wall A with velocity $2v_0$ on the rough surface with constant sliding friction to hit the wall B with velocity v_0. With what velocity same mass m should be projected to hit the wall B with same velocity v_0 if the surface is now moving upward with an acceleration of a = 4g?



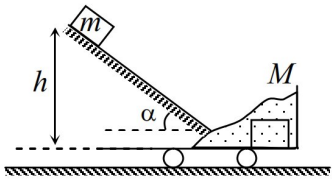
(A) $2v_0$
(B) $3\ v_0$
(C) $4\ v_0$
(D) $5\ v_0$

16. A particle of mass 1 kg is taken along the path ABCDE from A to E (see Figure). The two "hills" are of heights 50 m and 100 m and the horizontal distance AE is 20 m while the path length is 400 m. The coefficient of friction of the surface is 0.1. Take g = 10 m-s^{-2} and $\sqrt{3}$ = 1.73. The minimum work on the mass required to accomplish this is



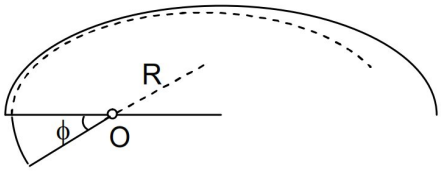
(A) 20 J
(B) 173 J
(C) 400 J
(D) 0 J

17. A block of mass *m* slides without friction down a fixed inclined board of inclination α with the horizontal. After leaving the inclined, the block falls on a cart of mass *M*. Initial height of the block above the level of the cart is *h* as shown. The velocity of cart when block drops on it will be



(A) $\dfrac{m\sqrt{2gh}}{M+m}$
(B) $\dfrac{m\sqrt{2gh}\sin\alpha}{M+m}$
(C) $\dfrac{m\sqrt{2gh}\cos\alpha}{M+m}$
(D) $\dfrac{m\sqrt{2gh}\cos\alpha}{M}$

18. A capacitor consists of two stationary plates shaped as a semi-circle of radius R and a movable plate made of dielectric with permittivity ε and capable of rotating about an axis O between the stationary plates. The thickness of the movable plate is equal to d which is practically the separation between the stationary plates. A potential difference V is applied to the capacitor. Find the magnitude of the moment of forces relative to the axis O acting on the movable plate in the position shown in the figure.

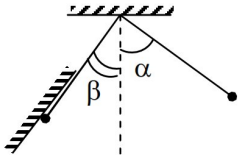


(A) $(\varepsilon-1)\dfrac{\varepsilon_0 R^2 v^2}{4d}$
(B) $(\varepsilon-1)\dfrac{\varepsilon_0 R^2 v^2}{2d}$
(C) $(\varepsilon-1)\dfrac{\varepsilon_0 R^2 v^2}{6d}$
(D) $(\varepsilon-1)\dfrac{\varepsilon_0 R^2 v^2}{8d}$

19. A charge $2\mu C$ is placed at each of the points $x = x_0, x = 3x_0, x = 5x_0, x = 7x_0 .....\infty$ on the x-axis and a charge $-2\mu C$ is placed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$, $x = 8x_0 .....\infty$ here $x_0 = 0.693$ m. The electric potential at the origin due to the above system is

(A) zero
(B) 9×10^3 V
(C) 18×10^3 V
(D) -9×10^3 V

20. A thin perfectly rigid weightless rod with a point-like ball fixed at one end is deflected through a small angle α from its equilibrium position and then released. At the moment when the rod forms an angle $\beta < \alpha$ with the vertical, the ball undergoes a perfectly elastic collision with an inclined wall (figure).



Determine the ratio T_1/T of the period of oscillations of this pendulum to the period of oscillations of a simple pendulum having the same length.

(A) $1-\frac{1}{\pi}\cos^{-1}\frac{\beta}{\alpha}$

(B) $1-\frac{2}{\pi}\cos\frac{\beta}{\alpha}$

(C) $1-\frac{1}{2\pi}\cos^{-1}\frac{\beta}{\alpha}$

(D) $1-\frac{1}{2\pi}\cos\frac{\beta}{\alpha}$

SECTION – B

(Numerical Answer Type)

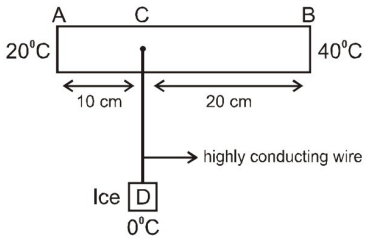
*This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.*

21. A thin uniform rod of mass 'm', length 'ℓ' rotates uniformly about a vertical axis ω, form a conical pendulum, the upper end of the rod is hinged and θ is angle between rod and vertical. If the change of angular momentum of the rod about hinge is $\frac{m\ell^2}{x}\omega^2\sin 2\theta$ then the value of 'x' will be

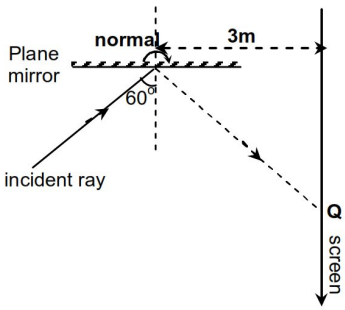
22. Binary stars rotate under mutual gravitational force at separation of $2\left(\frac{G}{\omega^2}\right)^{\frac{1}{3}}$ unit, where ω is the angular velocity of each of the star about centre of mass of the system. If difference between the mass of stars is 6 units. Find the ratio of masses of bigger star to smaller star.

23. A fixed container of height 'H' with large cross-sectional area 'A' is completely filled with water. Two small orifices of cross-sectional area 'a' are made, one at the bottom and the other on the vertical side of the container at a distance H/2 from the top of the container. The time taken by the water level to reach a height of H/2 from the bottom of the container is $\frac{XA}{3\alpha}\left(\sqrt{2}-1\right)\sqrt{\frac{H}{g}}$. Find the value of X.

24. In the figure shown AB is a rod of length 30cm and area of crossection 1.0 cm^2 and thermal conductivity 336 SI units. The ends A and B are maintained at temperature 20^0C and 40^0C respectively. A point C of this rod is connected to box D containing ice at 0^0C through highly conducting wire of negligible heat capacity. The rate at which ice melts in the box is *x* mg/s. The value of x/10 is (L_{ice} = 80 cal/gm)



25. In the given figure, a ray is incident on the plane mirror as shown. The reflected ray strike the screen at Q. the mirror now starts, rotating about axis passing through P and perpendicular to plane of paper with angular speed $\pi/180$ rad/second. If the speed of point at which reflected ray strike the screen is $\frac{k\pi}{15}$ m/second at time t = 15 second, find the value of k.



Chemistry

PART – B

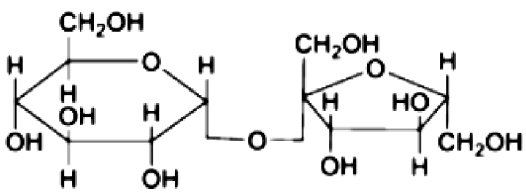
SECTION – A

(One Options Correct Type)

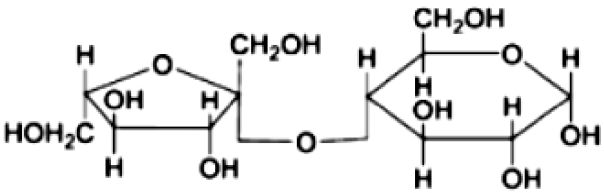
*This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.*

26. A disaccharide X cannot be oxidised by bromine water. The acid hydrolysis of X leads to a laevorotatory solution. The disaccharide X is

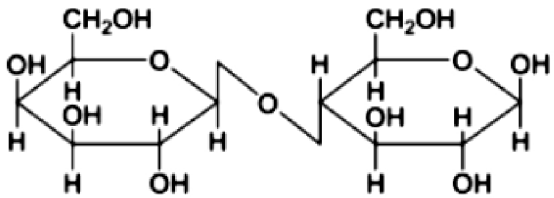
(A)



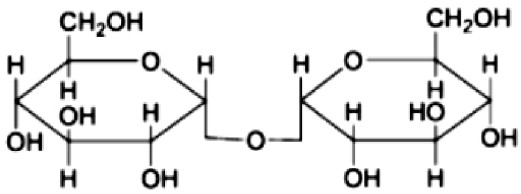
(B)



(C)



(D)



27. An alkaline (NaOH) solution of compound produces a yellow coloured solution on addition of $NaBO_3$. The compound is

(A) $Mn(OH)_2$
(B) $Pb(OH)_2$
(C) $Cr(OH)_3$
(D) $Fe(OH)_3$

28. The correct order of the number of unpaired electron in the given complexes is

A. $[Fe(CN)_6]^{3-}$
B. $[FeF_6]^{3-}$
C. $[CoF_6]^{3-}$
D. $[Cr(oxalate)_3]^{3-}$
E. $[Ni(CO)_4]$

Choose the correct answer from the options given below:

(A) A < E < D < C < B
(B) E < A < D < C < B
(C) E < A < B < D < C
(D) A < E < C < B < D

29. The products X and Y in the following reaction sequence, respectively, are

$$BCl_3 \xrightarrow[150\,^\circ C]{NH_4Cl \text{ in } C_6H_5Cl} X \xrightarrow{NaBH_4} Y$$

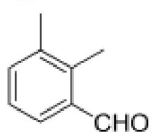
(A) $B_3N_3Cl_6$ and $B_3N_3H_6$
(B) $B_3N_3H_3Cl_3$ and $B_3N_3H_6$
(C) $B_3N_3H_3Cl_3$ and $B_3N_3H_{12}$
(D) $B_3N_3H_9Cl_3$ and $B_3N_3H_{12}$

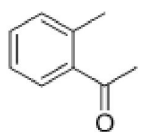
30. The role of fluorspar in the electrolytic reduction of Al_2O_3 is
(1) decrease the melting point of Al_2O_3
(2) improve the electrical conductivity of the melt
(3) prevent the corrosion of anode
(4) prevent the radiation loss of heat
(A) 1, 4
(B) 1, 2
(C) 1, 2,3, 4
(D) 1, 2, 4

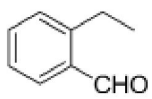
31. When concentrated HCl is added to an aqueous solution of $CoCl_2$, its colour changes from reddish pink to deep blue. Which complex ion gives blue colour in this reaction?
(A) $[CoCl_4]^{2-}$
(B) $[CoCl_6]^{3-}$
(C) $[CoCl_6]^{4-}$
(D) $[Co(H_2O)_6]^{2+}$

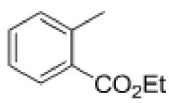
32. Which of the following elements have half-filled f-orbitals in their ground state?
(Given: atomic number Sm = 62; Eu = 63; Tb = 65; Gd = 64, Pm = 61)
A. Sm
B. Eu
C. Tb
D. Gd
E. Pm
Choose the correct answer from the options given below:
(A) B and D only
(B) A and E only
(C) A and B only
(D) C and D only

33. An organic compound on reaction with 2,4-dinitrophenylhydrazine (2,4-DNP) gives a yellow precipitate. It also gives silver mirror on reaction with ammoniacal AgNO3. It gives an alcohol and sodium salt of a carboxylic acid on reaction with concentrated NaOH. It yields benzene-1,2-dicarboxylic acid on heating with alkaline KMnO4. The structure of the compound among the following is

(A)


(B)


(C)


(D)


34. The pH of an aqueous buffer prepared using CH_3COOH and $CH_3COO^-\ Na^+$ IS 4.80.
The quantity $\frac{[CH_3COO^-]-[CH_3COOH]}{[CH_3COOH]}$ is_____
(round off to three decimal places
[Given: pK_a of CH_3COOH in water is 4.75]
Given $10^{0.05} = 1.1220$
(A) 0.122
(B) 0.125
(C) 0.120
(D) 0.123

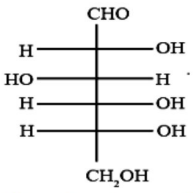
35. The (S°) of the following substances are
$CH_4(g)$ 186.2 JK^{-1} mol^{-1}; $O_2(g)$ 205.2 JK^{-1} mol^{-1}
$CO_2(g)$ 213.6 JK^{-1} mol^{-1}; $H_2O(l)$ 69.9 JK^{-1} mol^{-1}
The entropy change $(\Delta S°)$ for the reaction

$$CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(l)$$ is:

(A) –312.5 J K^{-1} mol^{-1}
(B) –242.8 J K^{-1} mol^{-1}
(C) –108.1 J K^{-1} mol^{-1}
(D) –37.6 J K^{-1} mol^{-1}

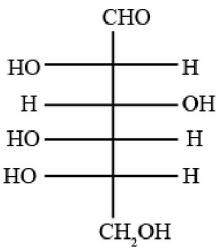
36. Higher order (>3) reactions are rare due to:
(A) shifting of equilibrium towards reactants due to elastic collisions
(B) loss of active species on collision
(C) low probability of simultaneous collision of all the reacting species
(D) increase in entropy and activation energy as more molecules are involved

37. The structure of D-glucose is

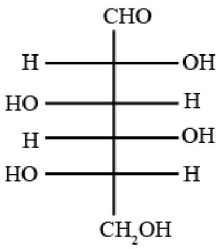


The structure of L-Glucose is

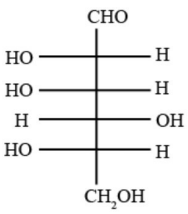
(A)



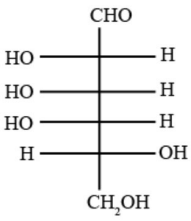
(B)



(C)

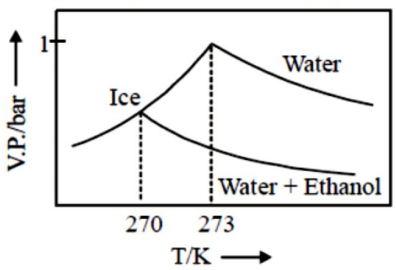


(D)

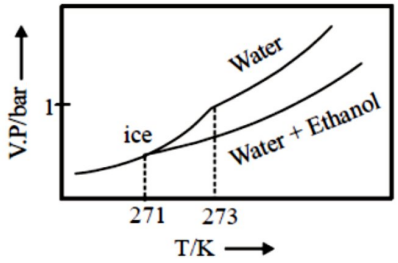


38. Pure water freezes at 273 K and 1 bar. The addition of 34.5 g of ethanol to 500 g of water changes the freezing point of the solution. Use the freezing point depression constant of water as 2 K kg mol^{-1}. The figures shown below represent plots of vapour pressure (V.P.) versus temperature (T). [molecular weight of ethanol is 46 g mol^{-1}] Among the following, the option representing change in the freezing point is

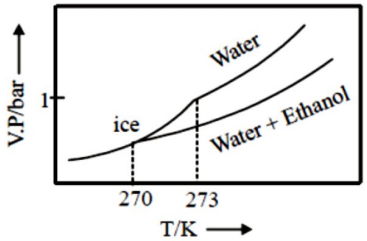
(A)



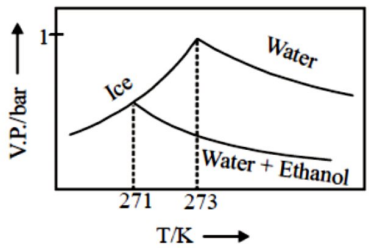
(B)



(C)



(D)



39. The Cl – C – Cl angle in 1, 1, 2, 2-tetrachloroethene and tetrachloromethane respectively will be about

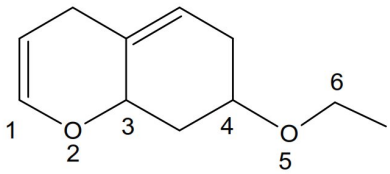
(A) 120º and 109.5º
(B) 90º and 109.5º
(C) 109.5º and 90º
(D) 109.5º and 120º

40. Sea water containing 1 M NaCl has to be desalinated at 300 K using a membrane permeable only to water. The minimum pressure (in bars) required on the sea-water side of the membrane is_____

($R = 8.3\ J\ mol^{-1}\ K^{-1}$, $1\ bar = 10^5\ N/m^2$)

(A) 24.94
(B) 49.88
(C) 2490
(D) 4980

41. On the treatment of the following compound with a strong acid, the most susceptible site for bond cleavage is



(A) O2 – C3
(B) O5 – C6
(C) C4 – O5
(D) C1 – O2

42. Among the following species, the one that has pentagonal shape is

(A) $XeOF_4$
(B) IF_5
(C) $[SF_5]^-$
(D) $[XeF_5]^-$

43. Consider the electrochemical cell

$M(s)\,|\,MI_2(s)\,|\,MI_2(aq)\,|\,M(s)$

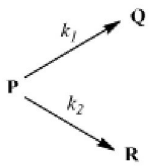
where 'M' is a metal. At 298 K, the standard reduction potential are

$E^0_{M^{2+}(aq)/M(s)} = -0.12\ V,\ E^0_{MI_2(s)/M(s)} = -0.36\ V$ and the temperature coefficient is

$\left(\frac{\partial E^0_{cell}}{\partial T}\right)_P = 1.5\times10^{-4}\ V\ K^{-1}$. At this temperature the standard enthalpy change for the overall cell reaction $\Delta_r H^0$, is_____kJ mol^{-1}

(A) –37.60 (B) 37.60
(C) 3.76 (D) –3.76

44. Consider the following two parallel irreversible first order reactions at temperature T



where k_1 and k_2 are the rate constants and their values are 5×10^{-2} and 15×10^{-2} min^{-1}, respectively, at temperature T. If the initial concentration of the reactant '**P**' is 4 mol L^{-1}, then the concentration of product '**R**' after 10 min of reaction is_____mol L^{-1}.

Given $e^{-2} = 0.135$

(A) 1.59 (B) 2.59
(C) 3.59 (D) 4.59

45. Gas phase bond length and dipole moment of a compound (MX) is 3 Å and 10.8 D, respectively. The ionic character in gas phase MX is_%.

($1D = 3.336 \times 10^{-30}$ C m)

(A) 64.50 (B) 74.50
(C) 84.50 (D) 90.50

SECTION – B

(Numerical Answer Type)

*This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.*

46. A certain orbital has n = 4 and $m_l = -3$. The number of radial nodes in this orbital is_____
(Round off to the Nearest Integer)

47. For the elementary reaction $C \xleftarrow{k_2} A \xrightarrow{k_1} B$, $k_1 = 2k_2$. At time t = 0, [A] = A_0 and [B] = [C] = 0. At a later time t, the value of [B]/[C] is_____
(round off to the nearest integer)

48. For the given general reactions

$P(g) \rightleftharpoons Q(g) \qquad K_C = 10$

$Q(g) \rightleftharpoons B(g) \qquad K_C = 2$

$B(g) \rightleftharpoons D(g) \qquad K_C = 0.01$

Calculate the value of K_C for the reaction $D(g) \rightleftharpoons P(g)$

49. The ratio of the 2p→ 1s transition energy in He^+ to that in the H atom is closest to

50. Total number of paramagnetic species NO, O_2, B_2 and C_2 in their ground state is

Mathematics

PART – C

SECTION – A

(One Options Correct Type)

*This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.*

51. If the coefficient of x^7 in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$ and coefficient of x^{-7} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$ are equal, then the minimum value of $a^2 + b^2$ is
(A) 0
(B) 1
(C) 2
(D) 3

52. If the circles $x^2 + y^2 + 10\alpha x + \beta y + \alpha = 0$ and $x^2 + y^2 - 5\alpha x + \gamma y - 1 = 0$ intersect in two distinct points A and B, then the line $15x + \delta y - \alpha = 0$ passes through A and B for
(A) infinity many values of α
(B) exactly two values of α
(C) exactly one value of α
(D) no value of α

53. If Z is a complex number such that $\arg\,(z(1+\bar{z})) + \arg\left(\frac{|z|^2}{z-|z^2|}\right) = 0$ then
(A) $\arg\ \bar{Z} = -\frac{\pi}{2}$
(B) $\arg z = \frac{\pi}{4}$
(C) $|\bar{z}| < 1$
(D) $\ln\left(\frac{1}{|z|}\right) \in (-\infty,\infty)$

54. The maximum value of $\left|\sqrt{x^2-3x^2-6x+13}-\sqrt{x^4+5x^2+4}\right|$ is
(A) 2
(B) 2
(C) 4
(D) 5

55. If $f(x)=\frac{x^3+x-2}{x^3-1}$ and $g(x)=\frac{x^2+x+2}{x^2+x+1}$ then (D_f represents domain and R_f represents range of f(x))
(A) $D_f = R-\{1\}\ R_f = \left(1,\frac{7}{3}\right]-\left\{\frac{4}{3}\right\}$
$D_g = R\ \ R_g = \left(1,\frac{7}{3}\right]$
(B) $D_f = R-\{1\}\ \ R_f = \left(1,\frac{7}{3}\right]$
$D_g = R\ \ R_g = \left[1,\frac{7}{3}\right]$
(C) $D_f = R-\{1\}\ \ R_f = \left[1,\frac{7}{3}\right]$
$D_g = R\ \ R_g = \left[1,\frac{7}{3}\right]$
(D) $D_f = R-\{1\}\ \ R_f = \left(1,\frac{7}{3}\right]$
$D_g = R\ \ R_g = \left(1,\frac{7}{3}\right]$

56. If f : {1, 2, 3, 4} → {1, 2, 3, 4} y = f(x) be a function such that $|f(\alpha)-\alpha| \le 1$, for $\alpha \in$ {1, 2, 3, 4} then total number of such functions are
(A) 81
(B) 36
(C) 54
(D) None of these

57. If $\sum_{n=1}^{\infty}\cot^{-1}\left(2+\frac{n(n+1)}{2}\right)=\tan^{-1}a$, then 'a' is equal to

(A) 1
(B) 2
(C) 3
(D) 4

58. The number of three-digit numbers abc, which satisfy $a \le b > c$ is

(A) $2({}^9C_2+{}^9C_3+{}^9C_1)$
(B) ${}^9C_2+{}^9C_3+{}^9C_1$
(C) $2({}^9C_2+{}^9C_3)+{}^9C_1$
(D) None of these

59. Let $\left[(5+\sqrt{b})^n\right]=N$ where b, n are natural number and $|5-\sqrt{b}|<1$. If b and n are picked randomly, then the probability that N is odd, belongs to set (where [.] denotes the greatest integer function)

(A) $\left(\frac{3}{4},1\right]$
(B) $\left\{\frac{3}{4}\right\}$
(C) $\left[\frac{1}{2},\frac{3}{4}\right)$
(D) None of these

60. Let One A.M. 'a' and two GMs g_1 and g_2 be inserted between 'b' and 'c' then $\frac{g_1^3+g_2^3}{abc}=$

(A) 2
(B) 1
(C) 3
(D) 4

61. a, b, c are positive integers forming an increasing G.P and b – a is a perfect cube and $\log_6 a + \log_6 b + \log_6 c = 6$, then a + b + c is equal to

(A) 100
(B) 111
(C) 122
(D) 189

62. The set of all possible values of parameter a such that the equation

$(1+a)\left(\frac{x^2}{1+x^2}\right)^2-3a\left(\frac{x^2}{1+x^2}\right)+4a=0$ has a real solution is

(A) $\left(-\frac{1}{2},0\right]$
(B) (–1, 1)
(C) $\left[-\frac{1}{2},\frac{1}{2}\right]$
(D) None of these

63. In ΔABC, X and Y be the foot of perpendicular drawn from A to the internal angle bisector of B and C. The slope of lines which makes angle 45º with the line XY are (Given slope of BC = 2)

(A) $2, -\frac{1}{2}$
(B) $-3, \frac{1}{3}$
(C) $4, -\frac{1}{4}$
(D) $5, -\frac{1}{5}$

64. The value of $\prod_{r=1}^{7}\cos\frac{\pi r}{15}$ is

(A) $\frac{1}{64}$
(B) $\frac{1}{128}$
(C) $\frac{1}{32}$
(D) $\frac{1}{4}$

65. If point A lines on $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and point B lies on $\frac{x-2}{3}=\frac{y-4}{7}=\frac{z-6}{6}$ then $\overline{AB}$ can not be parallel to

(A) i + j + 2k
(B) i + j + k
(C) i + j + 3k
(D) i + j + 4k

66. Let f(x) is cubic polynomial such that its local maximum is at (0, 1) and local minimum at (1, –2) and f(–1) < 0 and f(2) > 0. The value of $\sin^{-1}(\cos[\alpha])$ may be equal to
(where α is root of equation, and [.] represent greatest integer function

(A) $\frac{\pi}{2}-1$
(B) $\pi+2$
(C) $2-\pi$
(D) None of these

67. The maximum value of the function $f(x)=\frac{4\cot^{-1}x}{\pi}-\frac{\pi}{4\cot^{-1}(-x)}$ occurs at x equals to

(A) –1
(B) 0
(C) 1
(D) none of these

68. Area enclosed by y = g(x), x = 1 and x = 37, where g(x) is inverse of $f(x) = x^3 + 3x + 1$ is

(A) $\frac{91}{4}$
(B) $\frac{297}{4}$
(C) $\frac{207}{4}$
(D) None of these

69. Let points S_1 and S_2 (lying on positive x-axis) be the foci of the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{c^2}=\frac{1}{4}$. If a point of intersection of the two curves is equidistant from S_1 and S_2 and a : b = $\sqrt{7}:\sqrt{6}$ then the value of a : c is

(A) $\sqrt{7}:\sqrt{6}$
(B) $\sqrt{7}:\sqrt{18}$
(C) $\sqrt{7}:3$
(D) $\sqrt{7}:\sqrt{2}$

70. The mean and standard deviation of marks of 200 students were 40 and 15 respectively. Later it was discovered that a score of 40 was wrongly read as 50. The correct standard deviation is

(A) 39.95
(B) 14.98
(C) 224.5
(D) None of these

SECTION – B

(Numerical Answer Type)

*This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.*

71. Let f be a continuous and differentiable function in (x_1, x_2). If $f(x)\,f'(x) \geq x\sqrt{1-f((x))^4}$ and $\lim_{x\to x_1}(f(x))^2=1$ and $\lim_{x\to x_2^-}(f(x))^2=\frac{1}{2}$. Then minimum value of $\left[x_1^2-x_2^2\right]$ is .....(where [.] denotes the greatest integer function).

72. If $k = \lim_{x\to 1} \sec^{-1}\left(\frac{\lambda^2}{\ln x} - \frac{\lambda^2}{x-1}\right)$ exist, then the minimum value of [|λ|] is .....(where [.] denotes the greatest integer function)

73. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 7\hat{j} + 2\hat{k}$, $\vec{x}.\vec{a} = 0$, and $\vec{x}.\vec{c} = 0$ for some non-zero vector $\vec{x}$, then value of $\vec{a}.(\vec{b} \times \vec{c})$ is

74. The solution of $x^2dy - y^2dx + xy(x - y)dy = 0$ is $\ln\left|\frac{x-y}{xy}\right| = \frac{y^k}{2} + c$, then the value of k is

75. If $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x) f(t)dt$, then f(x) may be equal $\left(-\frac{1}{k}\sin x - \frac{2}{k}\cos x\right)$, where k is a numerical quantity which equals _____

SOLUTIONS

Physics

PART – A

SECTION – A

1. A

Sol. y_{cm} of solid hemisphere = 3R/8

y_{cm} of solid cone = R/4

2. A

Sol. $-\frac{1}{F} = P = 2P_{l_1} + 2P_{l_2} + P_m$ ...(1)

$P_{l_1} = \frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$P_{l_1} = [(1.5 - 1]\left[-\frac{1}{10} - \frac{1}{15}\right] = -\frac{1}{12}$ ...(2)

$P_{l_2} = \frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$P_{l_2} = \left(\frac{4}{3} - 1\right)\left[\frac{2}{15}\right] = \frac{2}{45}$ ...(3)

$P_m = -\frac{1}{f} = +\frac{2}{15}$ ...(4)

$-\frac{1}{F} = P = 2\left[-\frac{1}{12} + \frac{2}{45}\right] + \frac{2}{15} = -\frac{1}{6} + \frac{4}{45} + \frac{2}{15} = \frac{1}{18}$

$F = -18\,\text{cm}$. Focus is negative means system will behave as concave mirror.

3. D

Sol. Newton's equations are :

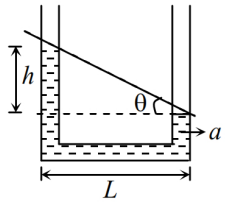
$A\Delta P\sin\theta = ma$ ...(i)

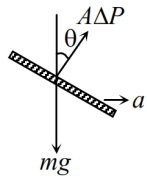
And $A\Delta P\cos\theta = mg$ ...(ii)

By (i) and (ii)

$\tan\theta = \frac{a}{g} = \frac{h}{L}$

or $h = \frac{aL}{g}$





4. A

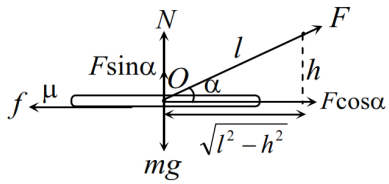
Sol. $N = mg - F\sin\alpha$

$F\cos\alpha = f = \mu N$

$F\cos\alpha = \mu(mg - F\sin\alpha)$

$$\mu = \frac{F\cos\alpha}{mg - F\sin\alpha} = \frac{F \times \frac{\sqrt{l^2 - h^2}}{l}}{mg - F \times \frac{h}{l}}$$

$$\mu = \frac{F\sqrt{l^2 - h^2}}{mgl - Fh}$$



5. C

Sol. Time taken her oscillation her any tunnel through earth is $\omega = \sqrt{\frac{g}{R}}$ consider earth tunnel and add total displacement.

$A \to S - \frac{\pi}{3}$

$S \to P - \frac{\pi}{2}$

$P \to C - \frac{\pi}{2}$

Total phase travelled $= \frac{4\pi}{3}$

$T = \frac{4\pi}{3\omega} = \frac{4\pi}{3}\sqrt{\frac{R}{g}}$

6. C

Sol. The net torque produced by the spring when the disc is rotated through an angle θ is

$\tau_C = (kx)R = (I_C + mr^2)\alpha$

Where $x = R\theta$ and $I_c = MR^2$

or $\alpha = \frac{kR^2}{\left(\frac{MR^2}{2} + mr^2\right)}\theta$ or $\omega = \sqrt{\frac{2kR^2}{MR^2 + 2mr^2}}$

7. D

Sol. $\left[\begin{array}{l}(33+a)(100)=(67+b)(50)\\(67+a)(50)=(33+b)(100)\end{array}\right.$

$$\left.\begin{array}{l}66+2a=67+b\\67+a=66+2b\end{array}\right]\times 2$$

$$65+3a=68$$

$a = 1$ and $b = 1$

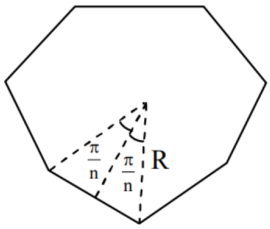
8. B

Sol. $U=-\vec{M}\cdot\vec{B}$

U is max when $\theta = 180°$

9. C

Sol.



$r = R\cos H/n$

B due to one side $=\dfrac{\mu_0 i}{4\pi r}\left[\sin\dfrac{\pi}{n}+\sin\dfrac{\pi}{n}\right]$

$$=\frac{\mu_0 i}{2\pi r}\sin\frac{\pi}{n}$$

$$=\frac{\mu_0 i}{2\pi r}\frac{\sin\pi/n}{R\cos\pi/n}$$

$$=\frac{\mu_0 i}{2\pi r}\tan\frac{\pi}{n}$$

$$B_{net}=\frac{n\mu_0 i}{2\pi R}\tan\frac{\pi}{n}$$

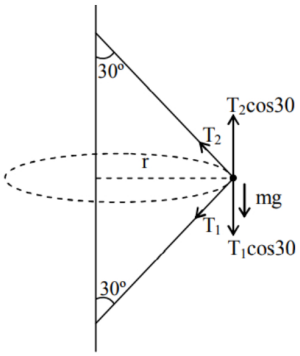
$$=\frac{\mu_0 i\tan\pi/n}{2R\frac{\pi}{n}}$$

10. D

Sol. Conceptual

11. A

Sol.



r = 0.4 sin 30
= 0.2 m

$$(T_1 + T_2)\sin 30 = \frac{200}{100}10^2 \times 0.2$$

$$\frac{T_1 + T_2}{2} = \frac{0.4}{10}\omega^2$$

$$T_1 \cos 30 + mg = T_2 \cos 30$$

$$T_1 \frac{\sqrt{3}}{2} + \frac{200}{100}10 = T_2 \frac{\sqrt{3}}{2}$$

$$\sqrt{3}\frac{(T_2 - T_1)}{2} = 2$$

$$T_2 - T_1 = \frac{4}{\sqrt{3}}$$

T_1 = 4N given

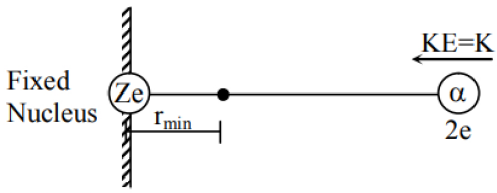
$$T_2 = 4 + \frac{4}{\sqrt{3}} = 6.26 \text{ N}$$

$$\frac{6.26 + 4}{2} = \frac{0.4}{10}\omega^2$$

$$\omega^2 = 128.25 = 11.32 \text{ rad/s}$$

12. C

Sol.



Mechanical energy conservation

$$0 + K = 0 + \frac{Ze \times 2e}{4\pi \in_0 r_{min}}$$

$$r_{min} = \frac{Ze^2 \times 2}{4\pi \in_0 K}$$

$r_{min} \propto Z$

So when Z become 2 Z r_{min} also get double so new r_{min} = 2b

13. A

Sol. $V_1 R_1 = V_2 R_2$

14. B

Sol. Conceptual

15. C

Sol. Conceptual

16. D

Sol. Conceptual

17. C

Sol. Speed of block at the bottom of board = $\sqrt{2gh}$

Applying conservation of linear momentum in horizontal direction,

$$m\sqrt{2gh}\cos\alpha = (M+m)v$$

$$v = \frac{m\sqrt{2gh}\cos\alpha}{M+m}$$

18. A

Sol. Conceptual

19. C

Sol. Conceptual

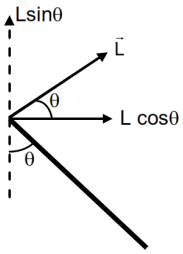
20. A

Sol. Conceptual

SECTION – B

21. 6

Sol. $\left|\frac{d\vec{L}}{dt}\right| = L\cos\theta\omega$



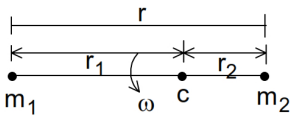
$$= \frac{m\ell^2}{3}\omega\sin\theta\cos\theta\omega$$

$$= \frac{m\ell^2}{3}\omega^2\sin\theta\cos\theta$$

22. 7

Sol. Given that $m_1 - m_2 = 6$ unit .....(i)

equation of motion



$$\frac{Gm_1m_2}{r^2} = m_1\omega^2 r_1 = m_1\omega^2\frac{m_2 r}{m_1+m_2}$$

$$\therefore m_1 + m_2 = \frac{\omega^2 r^3}{G} = 8 \text{ unit .....(ii)}$$

by (i) & (ii)

$$\frac{m_1}{m_2} = 7$$

23. 2

Sol. Conceptual

24. 4

Sol. Thermal resistance of AC $R_{AC} = \frac{L}{KA} = \frac{0.1}{336\times1\times10^{-4}}$

Thermal resistance of BC $R_{BC} = \frac{0.2}{336\times10^{-4}} = 2R$

Tc

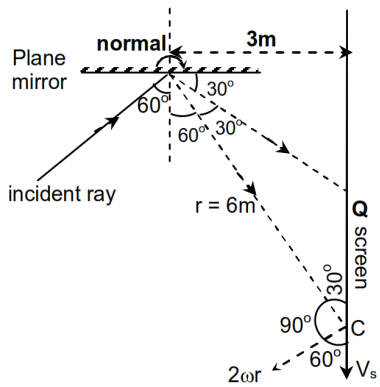
$$H_1 = \frac{20}{R}; H_2 = \frac{40}{2R} = \frac{20}{R}$$

$$H = H_1 + H_2 = \frac{40}{R} = 13.44\,W.$$

$$\text{Rate} = \frac{H}{Li} = \frac{13.44/4.2}{80}\ g/s = 40\ mg/sec.$$

25. 2

Sol. When incident ray is fixed the angular velocity of reflected ray becomes twice in the same sense as that of reflected ray. After t = 15 sec, the mirror rotates 15° then reflected ray rotates 30° in the same sense.



In ΔABC, $\cos 60° = \frac{3}{r}$

Where $\omega = \frac{\pi}{180} rad/sec. r = 6m$

Then, there are two components of velocity of spot on the screen.

(a) radial component which increase the length of r i.e.,

$$V_s \sin 60° = \frac{dr}{dt}$$

(b) perpendicular component

$$V_s \cos 60° \qquad \Rightarrow V_s \times \frac{1}{2} = 2 \times \frac{\pi}{180} \times 6 \Rightarrow V_s = \left(\frac{2\pi}{15} m/s\right)$$

Chemistry

PART – B

SECTION – A

26. C
Sol. Hydrolysis of sucrose brings about a change in the sign of rotation from dextro to laevo.

27. C
Sol. Factual

28. B
Sol. $\left[Fe(CN)_6\right]^{3-}, \left[Mn(CN)_6\right]^{3-}.\left[CoF_6\right]^{3-}, \left[MnBr_4\right]^{2-}$
have 1, 2, 4 & 5 unpaired electron respectively.

29. B
Sol. Factual

30. B
Sol. Factual

31. A
Sol. Tetrahedral complex has intense color.

32. A
Sol. Eu & Gd have half-filled f-orbitals (factual)

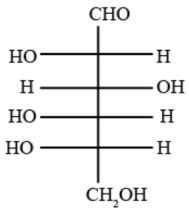
33. C
Sol.
- (2, 4-DNP) test given by aldehyde 1 ketones
- Tollen test
- Canizaro
- Must be at 1, 2 position

34. A
Sol. $pH = pK_a + \frac{\log\left[CH_3COO^-\right]}{CH_3COOH}$
C_p is temperature dependent

35. B
Sol. The entropy change for reaction in given by
$\Delta S^\circ_{reactant} = \Delta S^\circ_{product} - \Delta S^\circ_{reactant}$

36. C
Sol. Higher order reaction are rare due to low probability of simultaneous collision of all the reacting species.

37. A
Sol. The structure of L-Glucose is



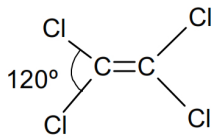
38. C

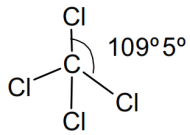
Sol. De pression in free ring point

$\Delta T_f = ik_{fm}$

39. A

Sol. Tetrachloroethene & tetrachloromethane





40. B

Sol. i = 2

π = iCRT

41. B

Sol. Most polar and weakest bond will break first.

42. D

Sol. VSEPR

43. A

Sol.
$$\begin{bmatrix} \Delta G^o = -nFE^o_{cell} \\ \Delta S^o = nF\left(\frac{\delta E^o_{cell}}{\delta T}\right) \\ \Delta H = \Delta G^o + T\Delta S^o \end{bmatrix}$$

44. B

Sol. $[R] = \frac{K_2}{K_1 + K_2}[P]\left(1 - e^{-(k_1+k_2)t}\right)$

45. B

Sol. % ionic character = $\frac{\text{observed dipole moment}}{\text{calculated dipole momet}} \times 100$

SECTION – B

46. 0

Sol. No. of radical nodes = n – l –1

= 4 –3 –1

= 0

47. 2

Sol. $\frac{[B]}{[C]} = \frac{k_1}{k_2} = \frac{2k_2}{k_2} = 2$

48. 5

Sol. K_{final} $P(g) \rightleftharpoons D(g)$ = $K_{C_1} \times K_{C_2} \times K_{C_3}$

$= 10 \times 2 \times 0.01 = \frac{20}{100} \times \frac{2}{10}$

= 1/5

$D \rightleftharpoons P(g)$

$k_f^1 = \frac{1}{k_{final}} = \frac{1}{1/5} = 5$

49. 4

Sol. $E_n = -\frac{13.5}{n^2} Z^2$

50. 3

Sol. Paramagnetic species are those which have unpaired of electron present in then NO, O_2 & B_2 have unpaired electron

SECTION – A

51. C

Sol. In expression $\left(ax^2+\frac{1}{bx}\right)^{11}$, we have

$T_{r+1}={}^{11}C_r\left(\frac{a^{11-r}}{b^r}\right)(x)^{22-3r}$ for x^7

$\Rightarrow 22-3r=7\Rightarrow r=5$

Also in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$

$t_{r+1}=(-1)^r\ {}^{11}C_r.\frac{a^{11-r}}{b^r}.x^{11-3r}$ for x^{-7}

$11-3r=-7\Rightarrow r=6$

Comparing, $T_6=t_7\Rightarrow ab=1$

Now $\frac{a^2+b^2}{2}\geq|ab|\Rightarrow a^2+b^2\geq 2$

52. D

Sol. Equation of common chord is $15\,\alpha x+(\beta+\gamma)\,y+(\alpha+1)=0$

Comparing with $15x+\delta y-\alpha=0$ passes through A and B for

$\therefore$ No real solution for α.

53. C

Sol. $\arg\left(z(1+\bar{z})\right)+\arg\left(\frac{z\bar{z}}{z(1-\bar{z})}\right)=0$

$\Rightarrow \arg z+\arg(1+\bar{z})+\arg\bar{z}-\arg(1-\bar{z})=0$

$\Rightarrow \arg\left(\frac{1+\bar{z}}{1-\bar{z}}\right)=0$

$\Rightarrow \frac{1+\bar{z}}{1-\bar{z}}=k>0$

$\Rightarrow \bar{z}=\frac{k-1}{k+1}$

$\Rightarrow |\bar{z}|<1$

54. D

Sol. $\left|\sqrt{(x^2-2)^2+(x-3)^2}-\sqrt{(x^2+2)^2+x^2}\right|$

i.e. $\left|\sqrt{(y-2)^2+(x-3)^2}-\sqrt{(y+2)^2+x^2}\right|$

Here $y=x^2$. If P(x, y) be any point on the parabola, then $|PA-PB|\leq AB$

Where A(3, 2), B(0, –2) $\Rightarrow$ AB = $\sqrt{9+16}=5$

55. D

Sol. $f(x)=\frac{x^3+x-2}{x^3-1}\Rightarrow f(x)=\left(\frac{x-1}{x-1}\right)\left(\frac{x^2+x+2}{x^2+x+1}\right)=\frac{x^2+x+2}{x^2+x+1}$, when $x\neq 1$

Hence range is $\left(1,\frac{7}{3}\right]$.

56. D

Sol. Total cases = $2 \times 3 \times 3 \times 2 = 36$

57. B

Sol. $t_n = \cot^{-1}\left(2 + \frac{n(n+1)}{2}\right)$

$$= \tan^{-1}\left(\frac{2}{4+n(n+1)}\right) = \tan^{-1}\left(\frac{\frac{1}{2}}{1+\frac{n}{2}\left(\frac{n+1}{2}\right)}\right) = \tan^{-1}\left(\frac{n+1}{2} - \tan^{-1}\frac{n}{2}\right)$$

$$\Rightarrow S_\infty = \frac{\pi}{2} - \tan^{-1}\frac{1}{2} = \cot^{-1}\frac{1}{2} = \tan^{-1}2$$

58. D

Sol. (i) $a \neq b \neq c$ all (distinct) and $c \neq 0$ — Total cases $2\,{}^9C_3$

(ii) $a \neq b$, $c = 0$ $(a = b > c)$ — Total cases ${}^9C_2$

(iii) $a = b$, $c \neq 0$ $(a = b > 0)$ — Total cases ${}^9C_2$

(iv) $a = b$, $c = 0$ $(a = b > 0)$ — Total cases ${}^9C_1$

(v) $a = c \neq 0$, $(a < b > c)$ — Total cases ${}^9C_2$

$2\,{}^9C_3 + 3\,{}^9C_2 + {}^9C_1$

59. C

Sol. If n = even $\Rightarrow$ N is odd

If n = odd N is even for b = 26, 27, ...35 and 25 and N is odd for b = 17, 18, ...24

60. A

Sol. $b + c = 2a$ and $g_1^3 = b^2c$

$g_2^3 = bc^2$

Now $\frac{g_1^3 + g_2^3}{abc} = 2$

61. D

Sol. $\text{Log}_6(abc) = 6$

$\Rightarrow (abc) = 6^6$

Let $a = \frac{b}{r}$ and $c = br$

$\Rightarrow b = 36$ and $a = \frac{36}{r} \Rightarrow r = 2, 3, 4, 6, 9, 12, 18$

Also, $36\left(1 - \frac{1}{r}\right)$ is a perfect cube.

$\Rightarrow 36\left(1 - \frac{1}{r}\right)$ is a perfect cube for r = 4.

$\Rightarrow a + b + c = 36 + 9 + 144 = 189.$

62. A

Sol. $1 + a = \frac{4 - 3z}{z^2 - 3z + 4}$

Where $z = \frac{x^2}{1+x^2}$; $\quad 0 \le z < 1$

Let $f(z)=\frac{4-3z}{z^2-3z+4}$, then $f'(z)=\frac{z(3z-8)}{(z^2-3z+4)^2}$

f(z) is decreasing in [0, 1), $\Rightarrow \frac{1}{2}<1+a\le 1;\ -\frac{1}{2}<a\le 0.$

63. B

Sol. XY is parallel to BC

64. B

Sol. $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}=\frac{1}{16}$

$\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}=\frac{1}{4}$

$\cos\frac{5\pi}{15}=\frac{1}{2}$

$\Rightarrow$ required value is $\frac{1}{128}$

65. A

Sol. $\overrightarrow{AB}$ can be in any direction but not in the plane of 2i + 3j + 4k and 3i + 7j + 6k since lines are skew.

66. A

Sol. When $[\alpha]$ = 0, 1, –1.

67. A

Sol. $f(x)=\frac{4}{\pi}(\pi-\cot^{-1}(-x))-\frac{\pi}{4\cot^{-1}(-x)}$

$=4-\left(\frac{4\cot^{-1}(-x)}{\pi}+\frac{\pi}{4\cot^{-1}(-x)}\right)\le 4-2=2$

Equality when $\cot^{-1}(-x)=\frac{\pi}{4}$

$\Rightarrow x=-1$

68. B

Sol. Area = $37\times 3-\int_0^3(x^3+3x+1)dx$

69. B

Sol. $S_1P=S_2P\Rightarrow a-e\alpha=E\alpha-\frac{a}{2}$. Also, $\alpha=\frac{ae+\frac{a}{2}E}{2}$

Eliminating α we get, $E^2+3eE+(2e^2-6)=0$

Now, as $e=\sqrt{1-\frac{b^2}{a^2}}=\frac{1}{\sqrt{7}}, E=\frac{5}{\sqrt{7}}\Rightarrow\frac{a}{c}=\frac{1}{\sqrt{E^2-1}}=\frac{\sqrt{7}}{\sqrt{18}}$

70. B

Sol. $\sum x = 40 \times 200 - 50 + 40 = 7990$

Corrected $\bar{x} = \frac{7990}{200} = 39.95$

Incorrect $\sum x^2 = n\left[\sigma^2 + \bar{x}^2\right] = 200\left[15^2 + 40^2\right] = 365000$

Correct $\sum x^2 = 36410 - 50^2 + 40^2 = 364100$

Correct $\sigma = \sqrt{\frac{364100}{200} - (39.95)^2} = 14.98$

SECTION – B

71. 1

Sol. $\frac{2f(x).f'(x)}{\sqrt{1-(f(x))^4}} - 2x \geq 0 \Rightarrow \frac{d}{dx}(\sin^{-1}(f(x))^2 - x^2) \geq 0$

Let $g(x) = \sin^{-1}((f(x))^2) - x^2$ is a non-decreasing function.

$\Rightarrow \lim_{x \to x_1} g(x) \leq \lim_{x \to x_2} g(x) \Rightarrow \frac{\pi}{2} - x_1^2 \leq \frac{\pi}{6} - x_2^2$

$\Rightarrow x_1^2 - x_2^2 \geq \frac{\pi}{3} \Rightarrow \left[x_1^2 - x_2^2\right] \geq 1.$

72. 1

Sol. $\lim_{x \to 1} \sec^{-1}\left(\frac{\lambda^2}{\ln x} - \frac{\lambda^2}{x-1}\right)$

Let $x - 1 = t \Rightarrow x = t + 1 \Rightarrow$ as $x \to 1$, $t \to 0$

$= \lim_{t \to 0} \sec^{-1}\left(\frac{\lambda^2}{\ln(t+1)} - \frac{\lambda^2}{t}\right) = \lim_{t \to 0} \sec^{-1}\left(\frac{\lambda^2}{t}\left(\frac{\lambda^2}{\ln(t+1)} - 1\right)\right)$

$= \lim_{t \to 0} \sec^{-1}\left(\frac{\lambda^2}{2}\right) \qquad \Rightarrow \frac{\lambda^2}{2} \geq 1 \,|\, \lambda \,| \geq \sqrt{2}$

Hence minimum value of $[|\lambda|] = 1$.

73. 0

Sol. Since $\vec{x}.\vec{a} = \vec{x}.\vec{b} = \vec{x}.\vec{c} = 0$

$\Rightarrow (\vec{a}, \vec{b}, \vec{c}$ are coplanar) $\vec{x}$ is perpendicular to $\vec{a}, \vec{b}, \vec{c}$

$\therefore \vec{a}.(\vec{b} \times \vec{c}) = 0$

74. 2

Sol. $x^2y^2\left(\frac{dy}{y^2} - \frac{dx}{x^2}\right) + x^2y^2\left(\frac{1}{y} - \frac{1}{x}\right)dy = 0$

$\Rightarrow d\left(\frac{1}{x} - \frac{1}{y}\right) + y\left(\frac{1}{y} - \frac{1}{x}\right)dy = 0 \Rightarrow \frac{d\left(\frac{1}{x} - \frac{1}{y}\right)}{\frac{1}{x} - \frac{1}{y}} = ydy$

$\Rightarrow \ln\left|\frac{1}{x} - \frac{1}{y}\right| = \frac{y^2}{2} + c \Rightarrow k = 2.$

75. 3

Sol. $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x)\left(-\frac{1}{k}\sin t - \frac{2}{k}\cos t\right)dt$

$= \sin x + I_1 + I_2 + I_3 + I_4$

Where $I_1 = -\frac{\sin x}{k}\int_{-\pi/2}^{\pi/2} \sin t\,dt = 0$

$I_2 = -\frac{2\sin x}{k}\int_{-\pi/2}^{\pi/2} \cos t\,dt = -\frac{4\sin x}{k}$

$I_3 = -\frac{\cos x}{k}\int_{-\pi/2}^{\pi/2} t\sin t\,dt = -\frac{2\cos x}{k}$

$I_4 = \frac{4\cos x}{k}\int_{-\pi/2}^{\pi/2} t\cos t\,dt = 0 \qquad (\because t\cos t \text{ is odd })$

$\Rightarrow -\frac{1}{k}\sin x - \frac{2}{k}\cos x = \sin x - \frac{4\sin x}{k} - \frac{2\cos x}{k}$

$\Rightarrow -\frac{1}{k} = 1 - \frac{4}{k} \Rightarrow k = 3.$